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APPLICATIONS OF GRACEFUL GRAPHS

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ABSTRACT

A simple graph G is an ordered pair of a non-empty set V of objects called vertices together with the set E of edges $e=uv$, $u,v \in V$, where $p = |V|$ and $q = |E|$. If $f: V \rightarrow \{0,1,\dots,q\}$ is an injective mapping and $f^*(uv) = |f(u) - f(v)| \forall uv \in E$. If $f^*(E) = \{1,2,\dots,q\}$ then f is called a graceful labeling of G and the graph which admits such numbering is called graceful graphs. This paper discuss the applications of graceful labeling of some graphs in radio-astronomy, development of missile guidance codes, spectral characterization of material using X-ray crystallography.

KEYWORDS: radio-astronomy, missile guidance codes, characterization, X-ray crystallography.

INTRODUCTION

A graph is defined to be comprised of two finite sets, the vertex set and the edge set, graph are often denoted by G and we write $G = (V(G), E(G))$ where $V(G)$ represents the vertex set (which is non-empty) and $E(G)$ represents the edge set (which may be empty). A vertex is simply an element of the vertex set and an edge represents a connection between two elements of an unordered pair from the vertex set.

A graph labeling is defined to be an assignment of integers to vertices or sometimes edges in a graph. Interest in such graph labelings has increased in the last few decades, giving rise to many different labeling. Significance of graph labelings is that they are used in many fields of study. Some common application for graph labeling are found in coding theory, X-ray crystallography, radar and missile guidance.

For a simple graph G , with $p = |V|$ and $q = |E|$, if $f: V \rightarrow \{0,1,\dots,q\}$ is injective mapping and $f^*(uv) = |f(u) - f(v)| \forall uv \in E$ and induced mapping $f^*(E) = \{1,2,\dots,q\}$ then f is called a Graceful labeling of G .

Coding Theory

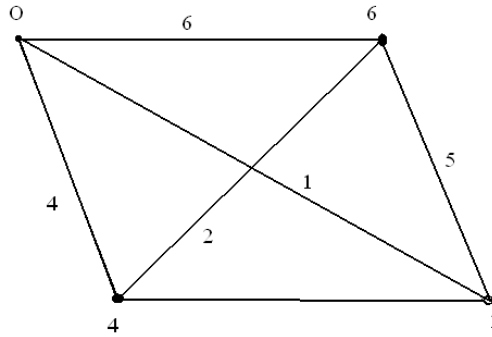
The design of certain important classes of good non periodic codes for pulse radar and missile guidance is equivalent to numbering of complete graph in such a way that all edge numbers are distinct. The vertex numbers then determine the time positions at which pulse are transmitted. Complete graph K_n is a graph in which every paired vertex are joined by an edge.

In complete graph K_4 ,

$$p = |V| = 4 \quad q = |E| = {}^4C_2 = 6$$

$$\therefore f: V \rightarrow \{0,1,\dots,6\} \quad f^*: E \rightarrow \{1,2,3,4,5,6\}$$

Such that



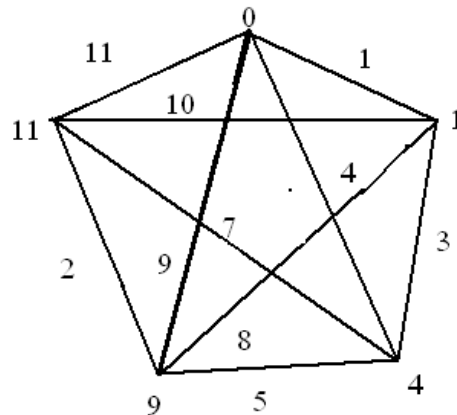
However K_5 is semi –graceful labeled, A semi –graceful labeling is defined to be one in which the constraint that the edge lengths need to be consecutive is relaxed , one edge length may be skipped by adding $n + 1$ edge length to the graph.

K_5 is semi –graceful labeled as follows.

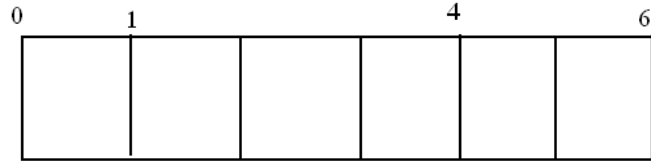
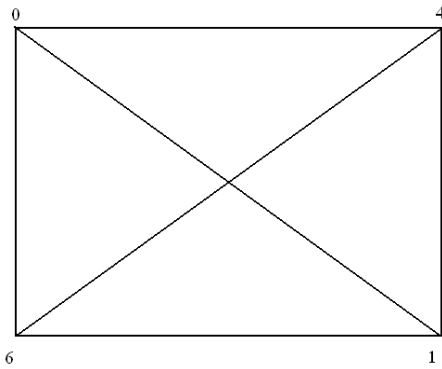
$$p = |V| = 5 \quad q = |E| = 10$$

$$f: V \rightarrow \{0,1,2, \dots, 11\}$$

$$f^*: E \rightarrow \{1,2,3,4,5,7,8,9,10,11\}$$



Quasi – graceful labeling is defined to be when the vertex labels are allowed to be extended beyond the largest edge length value however edge length constraints are left unchanged . Using these type of graceful labeling the extension to coding theory is made possible. Once the graph is gracefully , semi gracefully or quasi-gracefully labeled each vertex label is assigned to the ruler , while using no other tick mark on the ruler.
Ruler to the complete graph K_4



Communication Networks

If one had a communication network with a fixed number $n+1$ of communication centers (i.e. vertex) and they were numbered $0, 1, \dots, n$ then the lines between any two centers could be labeled with the difference between two center labels (i.e. vertex labels)

If the communication center grid was laid out in a graceful graph , we would then be able to label the connections between each center such that each connection would have a distinct label.

One good advantage of such a labeling is that if a link goes out , a simple algorithm could detect which two centers are no longer linked.

X – ray Crystallography

Position of atom in a crystal structures are made by X – ray diffraction patterns. Measurements indicate the set of inter atomic distances in crystal lattices.

Mathematically, one can find the finite set of integers $R = |0 = a_1 < a_2 < \dots < a_n|$ corresponds to one atom position , so that diffraction is equivalent to the distinct edge lengths (i.e. differences) between these two integers.

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